CHAPTER 9

Exercise Solutions

EXERCISE 9.1

From the equation for the AR(1) error model $e_t = \rho e_{t-1} + v_t$, we have

$$\text{var}(e_t) = \rho^2 \text{ var}(e_{t-1}) + \text{var}(v_t) + 2\rho\,\text{cov}(e_{t-1}, v_t)$$

from which we get

$$\sigma_e^2 = \rho^2 \sigma_e^2 + \sigma_v^2 + 0$$

$$\sigma_e^2 \left(1 - \rho^2\right) = \sigma_v^2$$

and hence

$$\sigma_e^2 = \frac{\sigma_v^2}{1 - \rho^2}$$

To find $E(e_t e_{t-1})$ we note that

$$e_t e_{t-1} = \rho e_{t-1}^2 + e_{t-1} v_t$$

Taking expectations,

$$E(e_t e_{t-1}) = \rho E\left(e_{t-1}^2\right) + 0 = \rho \sigma_e^2$$

Similarly,

$$e_t e_{t-2} = \rho e_{t-1} e_{t-2} + e_{t-2} v_t$$

and

$$E(e_t e_{t-2}) = \rho E(e_{t-1} e_{t-2}) + 0 = \rho E(e_t e_{t-1}) = \rho^2 \sigma_e^2$$

EXERCISE 9.2

(a) Using hand calculations

$$r_1 = \frac{\sum_{t=2}^{T} \hat{e}_t \hat{e}_{t-1}}{\sum_{t=2}^{T} \hat{e}_{t-1}^2} = \frac{0.0979}{0.5032} = 0.1946, \qquad r_2 = \frac{\sum_{t=3}^{T} \hat{e}_t \hat{e}_{t-2}}{\sum_{t=3}^{T} \hat{e}_{t-2}^2} = \frac{0.1008}{0.5023} = 0.2007$$

(b) (i) The test statistic for testing $H_0 : \rho_1 = 0$ against the alternative $H_1 : \rho_1 \neq 0$ is $Z = \sqrt{T} r_1 = \sqrt{10} \times 0.1946 = 0.6154$. Comparing this value to the critical Z values for a two tail test with a 5% level of significance, $Z_{(0.025)} = -1.96$ and $Z_{(0.975)} = 1.96$, we do not reject the null hypothesis and conclude that r_1 is not significantly different from zero.

(ii) The test statistic for testing $H_0 : \rho_2 = 0$ against the alternative $H_1 : \rho_2 \neq 0$ is $Z = \sqrt{T} r_2 = \sqrt{10} \times 0.2007 = 0.6347$. Comparing this value to the critical Z values for a two tail test with a 5% level of significance, $Z_{(0.025)} = -1.96$ and $Z_{(0.975)} = 1.96$, we do not reject the null hypothesis and conclude that r_2 is not significantly different from zero.

EXERCISE 9.3

(a) Equation (9.49) can be used to conduct two Lagrange multiplier tests for AR(1) errors. The first test is to test whether the coefficient for $\hat{e}_{t-1}$ is significantly different from zero. The null hypothesis is $H_0 : \rho = 0.$ The value of the test statistic is

$$t = \frac{0.428}{0.201} = 2.219$$

The critical t value for a 5% level of significance and 23 degrees of freedom is 2.069. Since 2.219 > 2.069, we reject the null hypothesis and conclude that first order autocorrelation is present.

The second *LM* test examines whether or not the R^2 from (9.49) is significant. The null hypothesis is again $H_0 : \rho = 0$ and the test statistic value is

$$LM = T \times R^2 = 26 \times 0.165 = 4.29$$

When the null hypothesis is true, *LM* has a $\chi^2_{(1)}$-distribution with a 5% critical value of 3.84. Since 4.29 > 3.84, we reject the null hypothesis and conclude that first order autoregressive errors exist.

(b) Ignoring autocorrelation means the estimates will be unbiased but no longer the best estimates. Furthermore, the standard errors will be incorrect resulting in misleading confidence intervals and hypothesis tests. The standard errors from the model with AR(1) errors are larger than the standard errors from the least-squares estimated model. Thus, it is likely that the least squares estimates and standard errors have overstated the precision of the estimates in the relationship between disposer shipments and durable goods expenditure. If autocorrelation is ignored, the confidence intervals will be narrower than the correct confidence intervals, and hypothesis tests will have a probability of a type 1 error that is greater than the specified significance level.

EXERCISE 9.4

(a) Under the assumptions of the AR(1) model, $\text{corr}(e_t, e_{t-k}) = \rho^k$. Thus,

(i) $\text{corr}(e_t, e_{t-1}) = \rho = 0.9$

(ii) $\text{corr}(e_t, e_{t-4}) = \rho^4 = 0.9^4 = 0.6561$

(iii) $\sigma_e^2 = \dfrac{\sigma_v^2}{1-\rho^2} = \dfrac{1}{1-0.9^2} = 5.263$

(b) (i) $\text{corr}(e_t, e_{t-1}) = \rho = 0.4$

(ii) $\text{corr}(e_t, e_{t-4}) = \rho^4 = 0.4^4 = 0.0256$

(iii) $\sigma_e^2 = \dfrac{\sigma_v^2}{1-\rho^2} = \dfrac{1}{1-0.4^2} = 1.190$

When the correlation between the current and previous period error is weaker, the correlations between the current error and the errors at more distant lags die out relatively quickly, as is illustrated by a comparison of $\rho_4 = 0.6561$ in part (a)(ii) with $\rho_4 = 0.0256$ in part (b)(ii). Also, the larger the correlation ρ, the greater the variance σ_e^2, as is illustrated by a comparison of $\sigma_e^2 = 5.263$ in part (a)(iii) with $\sigma_e^2 = 1.190$ in part (b)(iii).

EXERCISE 9.5

(a) (i) $\hat{e}_{T+1} = \rho e_T$

(ii) $\hat{e}_{T+2} = \rho \hat{e}_{T+1} = \rho^2 e_T$

(b) Equation (9.25) gives us the nonlinear least squares estimates of the coefficients $\hat{\beta}_1 = 3.89877$ and $\hat{\beta}_2 = 0.88837$. The final observation in *bangla.dat* is $A_{34} = 53.86$, $P_{34} = 0.89$. Therefore, the nonlinear least squares residual for the last observation is

$$\tilde{e}_{34} = \ln(53.86) - 3.89877 - 0.88837\ln(0.89) = 0.19114$$

(c) Forecasts for e_{T+1} and e_{T+2} are given by

$$\hat{e}_{T+1} = \hat{\rho}\tilde{e}_T = 0.42214 \times 0.19114 = 0.08069$$

$$\hat{e}_{T+2} = \hat{\rho}\hat{e}_{T+1} = 0.42214 \times 0.08069 = 0.03406$$

(d) Noting that $\hat{e}_{T+1} = \hat{\rho}\tilde{e}_T$, the forecast value for $\ln(A_{T+1})$ can be calculated as

$$\widehat{\ln(A_{T+1})} = \hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+1}) + \hat{\rho}\tilde{e}_T,$$
$$= 3.89877 + 0.88837\ln(1) + 0.08069 = 3.97946$$

Similarly, the forecast value for $\ln(A_{T+2})$ is

$$\widehat{\ln(A_{T+2})} = \hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+2}) + \hat{e}_{T+2}$$
$$= 3.89877 + 0.88837\ln(1.2) + 0.03406 = 4.09480$$

Exercise 9.5 (continued)

(e) In Chapter 4 we are told that there are two ways to forecast a dependent variable when the left-hand side of the equation is in the form of the logarithm of that variable. The first method is to calculate the "natural" predictor $\hat{y}_n$, which is the better predictor to use when the sample size is small

$$\hat{y}_n = \exp(b_1 + b_2 x)$$

The second method is to calculate a "corrected" predictor $\hat{y}_c$, which is the better predictor to use when the sample size is large

$$\hat{y}_c = \exp(b_1 + b_2 x + \hat{\sigma}^2/2) = \hat{y}_n e^{\hat{\sigma}^2/2}$$

Applied to our nonlinear least squares estimation, the natural predictors are

$$\begin{aligned}\hat{A}_{T+1} &= \exp\left(\hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+1}) + \hat{e}_{T+1}\right)\\ &= \exp\left(3.89877 + 0.88837\ln(1) + 0.08069\right)\\ &= 53.488\end{aligned}$$

$$\begin{aligned}\hat{A}_{T+2} &= \exp\left(\hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+2}) + \hat{e}_{T+2}\right)\\ &= \exp\left(3.89877 + 0.88837\ln(1.2) + 0.03406\right)\\ &= 60.027\end{aligned}$$

The corrected predictors are

$$\begin{aligned}\hat{A}_{T+1} &= \exp\left(\hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+1}) + \hat{e}_{T+1} + \hat{\sigma}_v^2/2\right)\\ &= \exp\left(3.89877 + 0.88837\ln(1) + 0.08069 + 0.2854^2/2\right)\\ &= 55.711\end{aligned}$$

$$\begin{aligned}\hat{A}_{T+2} &= \exp\left(\hat{\beta}_1 + \hat{\beta}_2 \ln(P_{T+2}) + \hat{e}_{T+2} + \hat{\sigma}_v^2/2\right)\\ &= \exp\left(3.89877 + 0.88837\ln(1.2) + 0.03406 + 0.2854^2/2\right)\\ &= 62.523\end{aligned}$$

EXERCISE 9.6

We consider two ways to derive the lag weights, by recursive substitution and by equating coefficients of the lag operator. Recursive substitution is tedious but does not require new machinery. Using the lag operator requires new machinery, but is less tedious.

Recursive substitution

One way to find the required expressions for the lag weights is to use recursive substitution on the ARDL model. Once we have substituted in enough lagged equations, we can determine the lag weights by observation. Recursive substitution begins with the current ARDL model

$$y_t = \delta + \delta_3 x_{t-3} + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + v_t \tag{1}$$

We lag the equation by 1 period

$$y_{t-1} = \delta + \delta_3 x_{t-4} + \theta_1 y_{t-2} + \theta_2 y_{t-3} + \theta_3 y_{t-4} + v_{t-1} \tag{2}$$

and then substitute it back into (1)

$$\begin{aligned} y_t &= \delta + \delta_3 x_{t-3} + \theta_1\left(\delta + \delta_3 x_{t-4} + \theta_1 y_{t-2} + \theta_2 y_{t-3} + \theta_3 y_{t-4} + v_{t-1}\right) \\ &\qquad + \theta_2 y_{t-2} + \theta_3 y_{t-3} + v_t \\ &= \delta + \delta_3 x_{t-3} + \theta_1\delta + \theta_1\delta_3 x_{t-4} + \theta_1^2 y_{t-2} + \theta_1\theta_2 y_{t-3} + \theta_1\theta_3 y_{t-4} \\ &\qquad + \theta_1 v_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + v_t \end{aligned} \tag{3}$$

This process is repeated with the larger lags until the required x_{t-s} is reached. In this model, we stop the process of recursive substitution after substituting in y_{t-3}. At this stage it should be clear that further substitution would not involve any additional lags of the independent variables x_{t-s} for $s = 1,2,3,4,5 \text{ or } 6$. This ensures that the expressions for the lag weights that we determine will not change with further substation. Rearranging the final equation should give an expression similar to

$$\begin{aligned} y_t &= \delta + \theta_1\delta + \theta_1^2\delta + \theta_1^3\delta + \theta_1\theta_2\delta + \theta_2\delta + \theta_1\theta_2\delta + \theta_3\delta \\ &\quad + \delta_3 x_{t-3} + \theta_1\delta_3 x_{t-4} + \theta_1^2\delta_3 x_{t-5} + \theta_2\delta_3 x_{t-5} + \theta_1^3\delta_3 x_{t-6} \\ &\quad + \theta_1\theta_2\delta_3 x_{t-6} + \theta_1\theta_2\delta_3 x_{t-6} + \theta_3\delta_3 x_{t-6} + \theta_1^4 y_{t-4} + \theta_1^2\theta_2 y_{t-4} \\ &\quad + \theta_1^2\theta_2 y_{t-4} + \theta_1^2\theta_2 y_{t-4} + \theta_1\theta_3 y_{t-4} + \theta_2^2 y_{t-4} + \theta_1\theta_3 y_{t-4} \\ &\quad + \theta_1^3\theta_2 y_{t-5} + \theta_1^3\theta_3 y_{t-6} + \theta_1^3 v_{t-3} + \theta_1^2\theta_3 y_{t-5} + \theta_1\theta_2^2 y_{t-5} \\ &\quad + \theta_1\theta_2^2 y_{t-5} + \theta_2\theta_3 y_{t-5} + \theta_2\theta_3 y_{t-5} + \theta_2\theta_3 y_{t-5} + \theta_1\theta_2\theta_3 y_{t-6} \\ &\quad + \theta_3^2 y_{t-6} + v_t + \theta_1 v_{t-1} + \theta_1^2 v_{t-2} + \theta_2 v_{t-2} + \theta_1\theta_2 v_{t-3} + \theta_1\theta_2 v_{t-3} + \theta_3 v_{t-3} \end{aligned} \tag{4}$$

By observation of (4) we can group the coefficients for each x_{t-s} remembering that β_s is defined as the coefficient of x_{t-s}. Therefore,

Exercise 9.6 (continued)

$$\beta_0 = \beta_1 = \beta_2 = 0$$
$$\beta_3 = \delta_3$$
$$\beta_4 = \theta_1\delta_3 = \theta_1\beta_3$$
$$\beta_5 = \theta_1^2\delta_3 + \theta_2\delta_3 = \theta_1\beta_4 + \theta_2\beta_3$$
$$\beta_6 = \theta_1^3\delta_3 + \theta_1\theta_2\delta_3 + \theta_1\theta_2\delta_3 = \theta_1\beta_5 + \theta_2\beta_4 + \theta_3\beta_3$$
$$= \theta_1\beta_{s-1} + \theta_2\beta_{s-2} + \theta_3\beta_{s-3}$$

Also the last result shows that

$$\beta_s = \theta_1\beta_{s-1} + \theta_2\beta_{s-2} + \theta_3\beta_{s-3},\ \text{for } s \ge 6$$

<u>Using the lag operator</u>

The lag operator L operates on a variable, say y_t, such that $Ly_t = y_{t-1}$. Although it will seem like magic to you at first, it is possible to do algebra with the lag operator. In particular, raising L to the power s, written L^s has the effect of lagging y_t s times. That is, $L^s y_t = y_{t-s}$. With this little bit of knowledge, the model

$$y_t = \delta + \delta_3 x_{t-3} + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3} + v_t$$

can be written as

$$(1 - \theta_1 L - \theta_2 L^2 - \theta_3 L^3) y_t = \delta + \delta_3 L^3 x_t + v_t$$

A bit more faith is required for the next step where we invert the left-hand side function of the lag operator to obtain

$$y_t = (1 - \theta_1 L - \theta_2 L^2 - \theta_3 L^3)^{-1}(\delta + \delta_3 L^3 x_t + v_t)$$

Now consider the infinite lag representation

$$y_t = \alpha + \sum_{s=0}^{\infty}\beta_s x_{t-s} + e_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \beta_4 x_{t-4} + \cdots + e_t$$

It can be written in terms of the lag operator as

$$y_t = \alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots) x_t + e_t$$

We now have two different equations for the same model, where y_t is the left-hand side variable for both of them. It follows that the right-hand sides must be equal

$$(1 - \theta_1 L - \theta_2 L^2 - \theta_3 L^3)^{-1}(\delta + \delta_3 L^3 x_t + v_t) = \alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots) x_t + e_t$$

and that

$$(\delta + \delta_3 L^3 x_t + v_t) = (1 - \theta_1 L - \theta_2 L^2 - \theta_3 L^3) \times \left[\alpha + (\beta_0 + \beta_1 L + \beta_2 L^2 + \beta_3 L^3 + \beta_4 L^4 + \cdots) x_t + e_t\right]$$

The left hand side can be written as

Exercise 9.6 (continued)

$$\delta + (0L^0 + 0L^1 + 0L^2 + \delta_3 L^3 + 0L^4 + 0L^5 + 0L^6)x_t + v_t$$

Although they have zero coefficients, we have included all powers of the lag operator up to lag 6. As we will see, it is useful to do so. Now consider the right hand side. After multiplying out and collecting terms, we have

$$\begin{aligned}(1-\theta_1-\theta_2-\theta_3)\alpha + \Big[&(\beta_0 L^0 + (\beta_1 - \theta_1\beta_0)L + (\beta_2 - \theta_2\beta_0 - -\theta_1\beta_1)L^2 \\ &+ (\beta_3 - \theta_3\beta_0 - \theta_2\beta_1 - \theta_1\beta_2)L^3 + (\beta_4 - \theta_3\beta_1 - \theta_2\beta_2 - \theta_1\beta_3)L^4 \\ &+ (\beta_5 - \theta_3\beta_2 - \theta_2\beta_3 - \theta_1\beta_4)L^5 + (\beta_6 - \theta_3\beta_3 - \theta_2\beta_4 - \theta_1\beta_5)L^6 + \cdots\Big]x_t \\ &+ (1-\theta_1 L - \theta_2 L^2 - \theta_3 L^3)e_t\end{aligned}$$

Equating coefficients of like terms establishes the relationship between the ARDL model and its infinite lag representation. For the constant and error term, we have

$$\delta = (1-\theta_1-\theta_2-\theta_3)\alpha$$

$$v_t = (1-\theta_1 L - \theta_2 L^2 - \theta_3 L^3)e_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$

For the lag weights we equate coefficients of equal powers of the lag operator

$$0 = \beta_0$$

$$0 = \beta_1 - \theta_1\beta_0$$

$$0 = \beta_2 - \theta_2\beta_0 - \theta_1\beta_1$$

$$\delta_3 = \beta_3 - \theta_3\beta_0 - \theta_2\beta_1 - \theta_1\beta_2$$

$$0 = \beta_4 - \theta_3\beta_1 - \theta_2\beta_2 - \theta_1\beta_3$$

$$0 = \beta_5 - \theta_3\beta_2 - \theta_2\beta_3 - \theta_1\beta_4$$

$$0 = \beta_6 - \theta_3\beta_3 - \theta_2\beta_4 - \theta_1\beta_5$$

From these expressions we obtain

$$\beta_0 = \beta_1 = \beta_2 = 0$$

$$\beta_3 = \delta_3$$

$$\beta_4 = \theta_1\beta_3$$

$$\beta_5 = \theta_2\beta_3 + \theta_1\beta_4$$

$$\beta_6 = \theta_3\beta_3 + \theta_2\beta_4 + \theta_1\beta_5$$

$$\beta_s = \theta_3\beta_{s-3} + \theta_2\beta_{s-2} + \theta_1\beta_{s-1} \quad \text{for } s \geq 6$$

For this process to work, and for the lag weights to be valid, coefficients for long lags must converge to zero.

The lag operator method will seem daunting at first, but it is worth the investment. You can go crazy doing recursive substitution.

EXERCISE 9.7

(a) The forecast *DIP* for January 2006 is

$$\begin{aligned}\widehat{DIP}_{T+1} &= 0.109 + 0.033DIP_T + 0.236DIP_{T-1} + 0.200DIP_{T-2} \\ &= 0.109 + 0.033\times 1.041 + 0.236\times 1.006 + 0.200\times 1.221 \\ &= 0.6250\end{aligned}$$

The forecast *DIP* for February 2006 is

$$\begin{aligned}\widehat{DIP}_{T+2} &= 0.109 + 0.033\widehat{DIP}_{T+1} + 0.236DIP_T + 0.200DIP_{T-1} \\ &= 0.109 + 0.033\times 0.625 + 0.236\times 1.041 + 0.200\times 1.006 \\ &= 0.5765\end{aligned}$$

The forecast *DIP* for March 2006 is

$$\begin{aligned}\widehat{DIP}_{T+3} &= 0.109 + 0.033\widehat{DIP}_{T+2} + 0.236\widehat{DIP}_{T+1} + 0.200DIP_T \\ &= 0.109 + 0.033\times 0.5765 + 0.236\times 0.625 + 0.200\times 1.041 \\ &= 0.4837\end{aligned}$$

(b) To find the 95% confidence intervals, we first find the forecast error standard errors using the expressions derived in Section 9.5

$$\hat{\sigma}_1 = \hat{\sigma}_v = 0.4293$$

$$\hat{\sigma}_2 = \hat{\sigma}_v\sqrt{1+\hat{\theta}_1^2} = 0.4293\sqrt{1+0.033^2} = 0.42953$$

$$\hat{\sigma}_3 = \hat{\sigma}_v\sqrt{\left(\hat{\theta}_1^2+\hat{\theta}_2\right)^2 + \hat{\theta}_1^2 + 1} = 0.4293\sqrt{\left(0.033^2+0.236\right)^2 + 0.033^2 + 1} = 0.44143$$

The model was estimated using monthly observations from January, 1985 to December, 2005, a total of 252 observations and 248 degrees of freedom. Confidence intervals were constructed using the *t*-value $t_{(0.975,248)} = 1.9696$.

The confidence intervals, given by the expression $\widehat{DIP}_{T+j} \pm \hat{\sigma}_j \times 1.9696$, are:

Month	$\widehat{DIP}_{T+j}$	$\hat{\sigma}_j$	Lower bound	Upper bound
January	0.6250	0.42930	−0.221	1.471
February	0.5765	0.42953	−0.269	1.422
March	0.4837	0.44143	−0.386	1.353

EXERCISE 9.8

Equation (9.28) is the estimated version of the model

$$y_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + \theta_1 y_{t-1} + v_t$$

In equation (9C.6) on page 266, expressions for the lag weights of the more general model

$$y_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + \delta_2 x_{t-2} + \delta_3 x_{t-3} + \theta_1 y_{t-1} + \theta_2 y_{t-2} + v_t$$

are given. Expressions for the lag weights for (9.28) can be obtained from the more general ones by setting $\delta_2 = \delta_3 = \theta_2 = 0$. Doing so yields

Weight	Estimate
$\beta_0 = \delta_0$	0.7766
$\beta_1 = \delta_1 + \theta_1\beta_0$	– 0.2969
$\beta_2 = \theta_1\beta_1$	– 0.1200
$\beta_3 = \theta_1\beta_2$	– 0.0485
$\beta_4 = \theta_1\beta_3$	– 0.0196
$\beta_5 = \theta_1\beta_4$	– 0.0079
$\beta_6 = \theta_1\beta_5$	– 0.0032

From the lag weight distribution, we can see that the immediate effect of a temporary 1% increase in the price of sugar cane is an increase in the area planted of 0.77%. In subsequent periods there are negative effects on the amount of area planted. One period after the temporary price increase the area planted decreases by 0.30%, the second period lagged effect is a decreases of 0.12%, the third period lagged effect is a decrease of 0.05%, the fourth period lagged effect is a decrease of 0.02%, and the fifth period lagged effect is a negligible decrease of less than 0.01%. This estimated lag weight distribution suggests that producers initially overreact to the price change. If the price increase was a sustained one, the final equilibrium change in area would be less than that which occurred in the current period.

Notice that we predict that producers will initially overreact to a price change because $\hat{\delta}_1 < -\hat{\theta}_1\hat{\beta}_0$. If $\delta_1 > -\theta_1\beta_0$, their initial response is one of under reaction. If $\delta_1 = -\theta_1\beta_0$, we have the AR(1) error model. There is no lagged response to price by itself, but there is a lagged response to the error in the previous period.

EXERCISE 9.9

(a) The least-squares estimated equation is

$$\widehat{\ln(JV_t)} = 3.503 - 1.612\ln(U_t)$$
$$\text{(se)} \quad (0.283) \quad (0.156)$$

A 95% confidence interval for β_2 is

$$b_2 \pm t_{(0.975,\,22)} \times \text{se}(b_2) = -1.6116 \pm 2.074 \times 0.1555 = (-1.934, -1.289)$$

(b) The *LM* test for $H_0 : \rho = 0$ can be conducted as a test for the significance of $\hat{\rho}$ from the equation

$$\hat{e}_t = \gamma_1 + \gamma_2 \ln(U_t) + \rho\hat{e}_{t-1} + \hat{v}_t$$

or by using the statistic $LM = T \times R^2$ from this equation. The value of the *F*-statistic for testing the significance of $\hat{\rho}$ is $F = 5.047$ with a *p*-value 0.036. Also, $LM = T \times R^2$ = $24 \times 0.19376 = 4.650$ with a *p*-value 0.031. Since both *p*-values are less than 0.05, we reject $H_0 : \rho = 0$ at a 5% significance level and conclude that autocorrelation exists. The existence of autocorrelation means the assumption that the e_t are independent is not correct. This problem causes the confidence interval for β_2 in part (a) to be incorrect; it could convey a false sense of the reliability of b_2

.

(c) The re-estimated model under the assumption of AR(1) errors is

$$\widehat{\ln(JV_t)} = 3.503 - 1.600\ln(U_t) \qquad \hat{\rho} = 0.4486$$
$$\text{(se)} \quad (0.249)\ (0.132) \qquad\qquad (0.2029)$$

The 95% confidence interval for β_2 is

$$b_2 \pm t_{(0.975,\,22)} \times \text{se}(b_2) = -1.6001 \pm 2.074 \times 0.1315 = (-1.873, -1.327)$$

This confidence interval is slightly narrower than that given in part (a). A direct comparison with the interval in part (a) is difficult because the least squares standard errors are incorrect in the presence of AR(1) errors. However, one could conjecture that the least squares confidence interval is larger than it should be implying unjustified imprecision.

Exercise 9.9(c) (continued)

The correlogram of residuals from the AR(1) error model is given below. To decide whether these correlations are significant, they are compared against the 5% significance bounds $\pm 1.96/\sqrt{24} = \pm 0.40$. None of the correlations exceed these bounds, suggesting that the AR(1) error model has successfully modeled the autocorrelation in the errors.

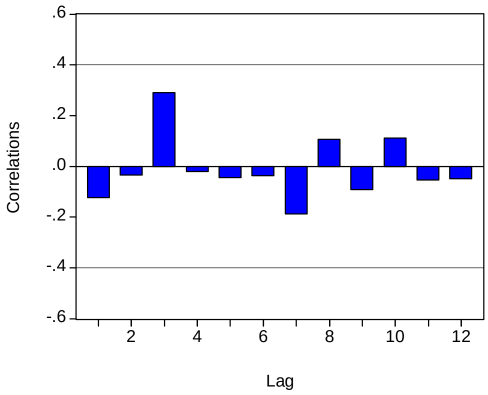


Figure xr9.9 Correlogram of residuals for Exercise 9.9 part (c)

EXERCISE 9.10

(a) The estimated equation with standard errors in parentheses is

$$\widehat{\ln(JV_t)} = 1.9438 - 1.6028\ln(U_t) + 0.7136\ln(U_{t-1}) + 0.4490\ln(JV_{t-1})$$
$$\text{(se)}\quad (0.8150)\ (0.1563)\qquad (0.3622)\qquad (0.2085)$$

Testing the hypothesis $H_0: \delta_1 = -\theta_1\delta_0$, we find the *F*-value to be 0.00115 with a corresponding *p* value of 0.9733. Since the *p*-value is greater than the level of significance 0.05, we do not reject the null hypothesis. Concluding that $\delta_1 = -\theta_1\delta_0$ implies the AR(1) error model is a reasonable one. The restricted model that incorporates the restriction $\delta_1 = -\theta_1\delta_0$ is

$$\ln(JV_t) = \delta + \delta_0 \ln(U_t) - \theta_1\delta_0 \ln(U_{t-1}) + \theta_1 \ln(JV_{t-1}) + v_t$$

This model is the transformed nonlinear least squares version of the AR(1) error model in Exercise 9.9.

(b) Forecasts for the next two periods are given by

$$\widehat{\ln(JV_{T+1})} = 1.9438 - 1.6028\ln(U_{T+1}) + 0.7136\ln(U_T) + 0.4490\ln(JV_T)$$
$$= 1.9438 - 1.6028\ln(5) + 0.7136\ln(3.72) + 0.4490\ln(5.12)$$
$$= 1.035$$

$$\widehat{\ln(JV_{T+2})} = 1.9438 - 1.6028\ln(U_{T+2}) + 0.7136\ln(U_{T+1}) + 0.4490\widehat{\ln(JV_{T+1})}$$
$$= 1.9438 - 1.6028\ln(5) + 0.7136\ln(5) + 0.4490\times 1.035$$
$$= 0.977$$

EXERCISE 9.11

(a) The marginal cost is the first derivative of the total cost with respect to quantity

$$\frac{dTC}{dQ} = \alpha_2 + 2\alpha_3 Q = MC$$

The marginal revenue is the first derivative of the total revenue with respect to quantity

$$\frac{dTR}{dQ} = \beta_1 + 2\beta_2 Q = MR$$

(b) To find the profit maximising quantity Q^*, equate the marginal revenue and the marginal cost

$$MR = MC \quad \Rightarrow \quad \beta_1 + 2\beta_2 Q^* = \alpha_2 + 2\alpha_3 Q^*$$

Rearranging so that Q^* is the subject gives

$$Q^* = \frac{\alpha_2 - \beta_1}{2(\beta_2 - \alpha_3)}$$

(c) The estimated least squares regression for the total revenue function with standard errors in parentheses is

$$\widehat{TR} = 174.2803Q - 0.5024Q^2$$
$$(4.5399) \quad (0.0235)$$

The estimated least squares regression for the total cost function with standard errors in parentheses is

$$\widehat{TC} = 2066.083 - 1.5784Q + 0.2277Q^2$$
$$(727.2180) \quad (9.4524) \quad (0.0289)$$

These estimates are appropriate under the multiple regression model assumptions. In particular, the error term must have an expected value of zero, $E(e_t) = 0$, must be homoskedastic, $\text{var}(e_t) = \sigma^2$, and the errors in different months must be uncorrealted, $\text{cov}(e_t, e_s) = 0$.

Using these estimates, the profit maximising level of output is

$$\hat{Q}^* = \frac{a_2 - b_1}{2(b_2 - a_3)} = \frac{-1.5784 - 174.2803}{2(-0.5024 - 0.2277)} = 120$$

Exercise 9.11 (continued)

(d) Assuming that the level of production for the next three months is based on the profit maximising level of output

$$\widehat{TR} = 174.2803 \times 120 - 0.5024 \times 120^2 = 13679$$

$$\widehat{TC} = 2066.083 - 1.5784 \times 120 + 0.2277 \times 120^2 = 5155$$

$$\widehat{PROFIT} = \widehat{TR} - \widehat{TC} = 13679 - 5155 = 8524$$

Therefore, for the next three months we forecast total revenue to be 13,679, the total cost to be 5,155, and the profit per month as 8,524.

(e) Performing *LM* tests for AR(1) errors in the total cost function yields $F = 8.553$ (*p*-value = 0.0054) on the significance of $\hat{\rho}$ and $LM = T \times R^2 = 7.812$ (*p*-value = 0.0052) using the χ^2 statistic. The same tests for AR(1) errors in the total revenue function yield $F = 113$ (*p*-value = 0.0000) on the significance of $\hat{\rho}$ and $LM = T \times R^2 = 34.4$ (*p*-value = 0.0000) using the χ^2 statistic. We conclude that both functions have correlated errors.

Examination of the correlograms of the residuals confirms this conclusion. The significance bounds in the figures below are at $\pm 1.96 / \sqrt{48} = \pm 0.283$. We find that there are several statistically significant correlations that exceed these bounds. In particular, r_1 of the total cost model and r_1, r_2, r_3 and r_4 of the total revenue model are statistically significant. They also lead us to conclude that the errors are correlated.

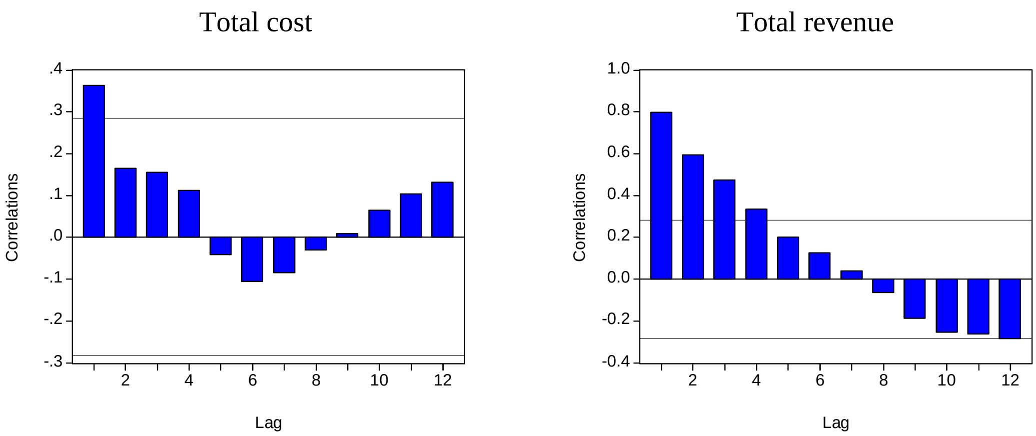


Figure xr9.11 Residual correlograms for total cost and total revenue functions

Exercise 9.11 (continued)

(f) The re-estimated equations assuming AR(1) errors are

$$\widehat{TR} = 171.58Q - 0.5085Q^2 \qquad e_t = 0.9495e_{t-1} + v_t$$
$$\text{(se)} \quad (8.01) \quad (0.0248) \qquad\qquad (0.0634)$$

$$\widehat{TC} = 2354.71 - 5.7521Q + 0.2415Q^2 \qquad e_t = 0.4595e_{t-1} + v_t$$
$$\text{(se)} \quad (616.01)\ (8.0203) \quad (0.0247) \qquad\qquad (0.1521)$$

(g) The profit maximising level of output suggested by the results in part (f) is

$$\hat{Q}^* = \frac{a_2 - b_1}{2(b_2 - a_3)} = \frac{-5.7521 - 171.5814}{2(-0.5085 - 0.2415)} = 118$$

(h) In this case, because the errors are assumed autocorrelated, the total revenue and total cost errors for month 48 have a bearing on the predictions, and the predictions will be different in each of the future three months.

For the total revenue function, the estimated error for month 48 is

$$\tilde{e}_{TR,48} = 8435 - \left(171.5814 \times 83 - 0.5085 \times (83)^2\right) = -2303.29$$

Therefore, given Q = 118 for the next three months, the total revenue predictions for the next three months are given by

$$\widehat{TR}_{T+1} = \widehat{TR}_{49} = 171.5814 \times 118 - 0.5085 \times 118^2 + 0.9495 \times (-2303.29) = 10979$$

$$\widehat{TR}_{T+2} = \widehat{TR}_{50} = 171.5814 \times 118 - 0.5085 \times 118^2 + 0.9495^2 \times (-2303.29) = 11090$$

$$\widehat{TR}_{T+3} = \widehat{TR}_{51} = 171.5814 \times 118 - 0.5085 \times 118^2 + 0.9495^3 \times (-2303.29) = 11195$$

For the total cost function, the error for the last sample month is

$$\tilde{e}_{TC,48} = 4829 - (2354.71 - 5.7521(83) + 0.2415(83^2)) = 1287.71$$

Therefore, given Q = 118 for the next three months, the total cost predictions for the next three months are given by

$$\widehat{TC}_{T+1} = \widehat{TC}_{49} = 2354.71 - 5.7521 \times 118 + 0.2415 \times 118^2 + 0.4595 \times 1287.71 = 5631$$

$$\widehat{TC}_{T+2} = \widehat{TC}_{50} = 2354.71 - 5.7521 \times 118 + 0.2415 \times 118^2 + 0.4595^2 \times 1287.71 = 5311$$

$$\widehat{TC}_{T+3} = \widehat{TC}_{51} = 2354.71 - 5.7521 \times 118 + 0.2415 \times 118^2 + 0.4595^3 \times 1287.71 = 5164$$

Exercise 9.11(h) (continued)

Profits for the months of 49, 50 and 51 (obtained by subtracting total cost from total revenue) are

$$\widehat{PROFITS}_{49} = 5349 \qquad \widehat{PROFITS}_{50} = 5779 \qquad \widehat{PROFITS}_{51} = 6031$$

Because $\tilde{e}_{TR,48}$ is negative, and its impact declines as we predict further into the future, the total revenue predictions become larger the further into the future we predict. The opposite happens with total cost; it declines because $\tilde{e}_{TC,48}$ is positive. Combining these two influences means that the predictions for profit increase over time. These predictions are, however, much lower than 8524, the prediction for profit that was obtained when autocorrelation was ignored. Thus, even although autocorrelation has little impact on the optimal setting for Q^*, it has considerable impact on the predictions of profit. This impact is caused by a change in the coefficient estimates, a relatively large negative residual for revenue in month 48, and a relatively large positive residual for cost in month 48.

EXERCISE 9.12

(a) The estimated model with standard errors in parentheses is

$$\widehat{\ln(A_t)} = 3.8241 + 0.7746\ln(P_t) - 0.2175\ln(P_{t-1}) - 0.0026\ln(P_{t-2})$$
$$\text{(se)} \quad (0.1006)(0.3129) \qquad (0.3185) \qquad (0.3221)$$

$$+ 0.5868\ln(P_{t-3}) - 0.0143\ln(P_{t-4})$$
$$(0.3153) \qquad (0.2985)$$

	Multipliers	
Lag	Delay	Interim
0	0.7746	0.7746
1	– 0.2175	0.5572
2	– 0.0026	0.5546
3	0.5868	1.1414
4	– 0.0143	1.1271

Only b_0, the coefficient of $\ln(P_t)$, is significantly different from zero at a 5% level of significance. All coefficients for lagged values of $\ln(P_t)$, namely, b_1, b_2, b_3, b_4, are not significant at a 5% level. This result is symptomatic of collinearity in the data. When collinearity exists, least squares cannot distinguish between the individual effects of each independent variable, resulting in large standard errors and coefficients which are not significantly different from zero.

Interpreting the delay multipliers, if the price is increased and then decreased by 1% in period *t*, there is an immediate increase of 0.77% in area planted. In period $t+1$, that is one period after the price shock, there is a decrease in area planted of 0.22%. In period $t+2$ there is practically no change in the area planted. In period $t+3$ there is an increase in area planted by 0.59% and in period $t+4$ there is a decrease of 0.01%.

The interim multipliers represent the full effect in period $t+s$ of a sustained 1% increase in price in period *t*. Thus, if the price increases by 1% in period *t*, there is an immediate increase in the area planted of 0.77%. The total increase when period $t+1$ is reached is 0.56%, at period $t+2$ it is 0.55%, at period $t+3$ it is 1.14% and, after $t+4$ periods there is a 1.13% increase.

The different signs attached to the delay multipliers, the relatively large weight at $t-3$, and the interim multipliers that decrease and then increase are not realistic for this example. The pattern is likely attributable to imprecise estimation.

Exercise 9.12 (continued)

(b) Using the straight line formula the lag weights are

$$\beta_0 = \alpha_0 \qquad i = 0$$
$$\beta_1 = \alpha_0 + \alpha_1 \qquad i = 1$$
$$\beta_2 = \alpha_0 + 2\alpha_1 \qquad i = 2$$
$$\beta_3 = \alpha_0 + 3\alpha_1 \qquad i = 3$$
$$\beta_4 = \alpha_0 + 4\alpha_1 \qquad i = 4$$

Substituting these weights into the original model gives

$$\begin{aligned}\ln(A_t) &= \alpha + \alpha_0 \ln(P_t) + (\alpha_0 + \alpha_1)\ln(P_{t-1}) + (\alpha_0 + 2\alpha_1)\ln(P_{t-2}) \\ &\qquad + (\alpha_0 + 3\alpha_1)\ln(P_{t-3}) + (\alpha_0 + 4\alpha_1)\ln(P_{t-4}) + e_t \\ &= \alpha + \alpha_0\left(\ln(P_t) + \ln(P_{t-1}) + \ln(P_{t-2}) + \ln(P_{t-3}) + \ln(P_{t-4})\right) \\ &\qquad + \alpha_1\left(\ln(P_{t-1}) + 2\ln(P_{t-2}) + 3\ln(P_{t-3}) + 4\ln(P_{t-4})\right) + e_t \\ &= \alpha + \alpha_0 z_{t0} + \alpha_1 z_{t1} + e_t\end{aligned}$$

(c) The estimated equation with standard errors in parentheses is

$$\widehat{\ln(A_t)} = 3.8266 + 0.4247 z_{t0} - 0.0996 z_{t1}$$
$$\text{(se)} \quad (0.1056) \; (0.2594) \quad (0.1088)$$

The least squares estimates of α_0 and α_1 are 0.4247 and -0.0996 respectively.

(d) The estimated weights are

$$\hat{\beta}_0 = a_0 = 0.42467$$
$$\hat{\beta}_1 = a_0 + a_1 = 0.42467 - 0.09963 = 0.3250$$
$$\hat{\beta}_2 = a_0 + 2a_1 = 0.42467 - 2 \times 0.09963 = 0.2254$$
$$\hat{\beta}_3 = a_0 + 3a_1 = 0.42467 - 3 \times 0.09963 = 0.1258$$
$$\hat{\beta}_4 = a_0 + 4a_1 = 0.42467 - 4 \times 0.09963 = 0.0261$$

These lag weights satisfy expectations as they are positive and diminish in magnitude as the lag length increases. They imply that the adjustment to a sustained price change takes place gradually, with the biggest impact being felt immediately and with a declining impact being felt in subsequent periods. The linear constraint has fixed the original problem where the signs and magnitudes of the lag weights varied unexpectedly.

Exercise 9.12 (continued)

(e) The new multipliers are

	Multipliers	
Lag	Delay	Interim
0	0.4247	0.4247
1	0.3250	0.7497
2	0.2254	0.9751
3	0.1258	1.1009
4	0.0261	1.1270

These delay multipliers are all positive and steadily decrease as the lag becomes more distant. This result, compared to the positive and negative multipliers obtained earlier, is a more reasonable one. It is interesting that the total effect, given by the 4-year interim multiplier, is almost identical in both cases, and the 3-year interim multipliers are very similar. The earlier interim multipliers are quite different however, with the restricted weights leading to a smaller initial impact.

EXERCISE 9.13

(a) The estimated equation with standard errors in parentheses is

$$\hat{y}_t = 1.0942 - 12.2998x_t - 27.0635x_{t-1} - 0.4001y_{t-1}$$
$$(5.7824)\ (26.6145)\quad (26.4483)\quad (0.0601)$$

(i) $\hat{y}_t$ is the predicted change in housing starts (in thousands) from month $t-1$ to month t and x_t is the change in interest rates from month $t-1$ to month t. The estimated coefficients of x_t and x_{t-1} are negative and suggest that the immediate effect of a temporary one unit increase in x_t is a decrease in y_t of 12.30 and the one month lagged effect is a decrease in y_t of 27.06. The coefficient of y_{t-1} is negative, suggesting that a positive change in housing starts lead to a negative change in housing starts in the following period. These signs are generally in line with our expectations. However, more implications of the signs and magnitudes of the coefficients can be obtained by examining the lag weights in the infinite lag representations as is done later in the question. The only coefficient estimate that is significantly different from zero is $\hat{\theta}_1$. Thus, with two important coefficients not significantly different from zero (those for x_t and x_{t-1}), the model is not a reasonable one.

(ii) Testing the hypothesis $H_0: \delta_1 = -\theta_1\delta_0$ against the alternative $H_1: \delta_1 \neq -\theta_1\delta_0$ delivers an F test value of 0.483861 with a p-value of 0.4873. Since the p value is greater that the 0.05 level of significance, we do not reject the null hypothesis. On the basis of this test, a restricted model of the form $y_t = \beta_1 + \beta_2 x_t + e_t$ with AR(1) errors is reasonable.

(iii) A correlogram of the residuals is presented below. Significance bounds are drawn at $\pm 1.96/\sqrt{250} = \pm 0.124$. Although not large, the correlations r_2, r_{12} and r_{24} are statistically significant. Since the data are monthly, there could be annual effect that is not being picked up. Overall, given these correlations and the insignificant coefficients mentioned in part (a), further modeling is in order.

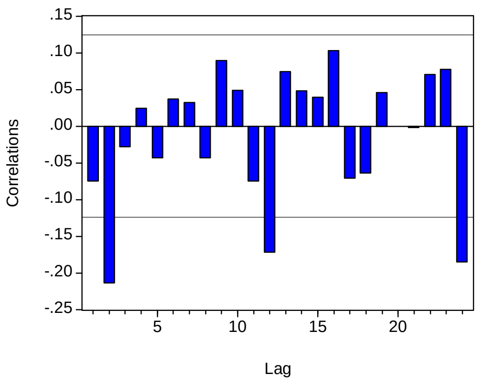


Figure xr9.13a Residual correlogram for Exercise 9.13 part (a)

Exercise 9.13 (continued)

(b) The estimated model with standard errors in parentheses is

$$\hat{y}_t = \underset{(5.5671)}{0.4856} - \underset{(23.8935)}{58.4292} x_{t-3} - \underset{(0.0631)}{0.5227} y_{t-1} - \underset{(0.0684)}{0.2903} y_{t-2} - \underset{(0.0634)}{0.1641} y_{t-3}$$

(i) In contrast to the model estimated in part (a) all the estimated coefficients are significant (with the exception of the intercept). There is a 3-month lagged effect on housing starts from an initial interest rate change, and the effect is negative as one would expect. The implications of the signs and magnitudes of the lagged *y* variables are better assessed from the consequent estimates of the lagged weights.

(ii) Using the results from Exercise 9.6 we obtain the following lag weights

s	Lag weight estimate	
0	$\hat{\beta}_0$	0
1	$\hat{\beta}_1$	0
2	$\hat{\beta}_2$	0
3	$\hat{\beta}_3 = \hat{\delta}_3$	– 58.4292
4	$\hat{\beta}_4 = \hat{\theta}_1\hat{\beta}_3$	30.5396
5	$\hat{\beta}_5 = \hat{\theta}_1\hat{\beta}_4 + \hat{\theta}_2\hat{\beta}_3$	1.0023
6	$\hat{\beta}_6 = \hat{\theta}_1\hat{\beta}_5 + \hat{\theta}_2\hat{\beta}_4 + \hat{\theta}_3\hat{\beta}_3$	0.1963
7	$\hat{\beta}_7 = \hat{\theta}_1\hat{\beta}_6 + \hat{\theta}_2\hat{\beta}_5 + \hat{\theta}_3\hat{\beta}_4$	– 5.4046
8	$\hat{\beta}_8 = \hat{\theta}_1\hat{\beta}_7 + \hat{\theta}_2\hat{\beta}_6 + \hat{\theta}_3\hat{\beta}_5$	2.6034
9	$\hat{\beta}_9 = \hat{\theta}_1\hat{\beta}_8 + \hat{\theta}_2\hat{\beta}_7 + \hat{\theta}_3\hat{\beta}_6$	0.1762
10	$\hat{\beta}_{10} = \hat{\theta}_1\hat{\beta}_9 + \hat{\theta}_2\hat{\beta}_8 + \hat{\theta}_3\hat{\beta}_7$	0.0388
11	$\hat{\beta}_{11} = \hat{\theta}_1\hat{\beta}_{10} + \hat{\theta}_2\hat{\beta}_9 + \hat{\theta}_3\hat{\beta}_8$	– 0.4986
12	$\hat{\beta}_{12} = \hat{\theta}_1\hat{\beta}_{11} + \hat{\theta}_2\hat{\beta}_{10} + \hat{\theta}_3\hat{\beta}_9$	0.2204

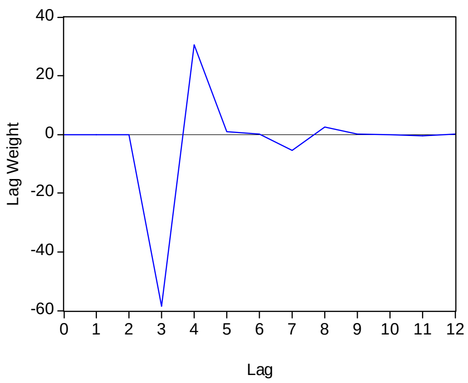


Figure xr9.13b Lag weights for Exercise 9.13 part (b)

Exercise 9.13(b)(ii) (continued)

(b) (ii) Figure 9.5 gives us a clear depiction of the distributed lag weights. The first three lag weights are zero indicating that interest rates have no effect on changes in housing starts over the two months following a rate change. The major impact of interest rates appears to follow at lags 3 and 4. A change in interest rates has the largest negative effect on housing starts at lag 3, and a positive readjustment occurs at lag 4. From lag 5 onwards, the effect on housings starts is relatively insignificant with the exception of the small spikes at lags 7 and 8.

(iii) A correlogram of the residuals is presented in Figure xr9.13c. The new model has eliminated the significant correlation at lag 2 that we found in the earlier model, but those at lags 12 and 24 remain, although they are small in magnitude.

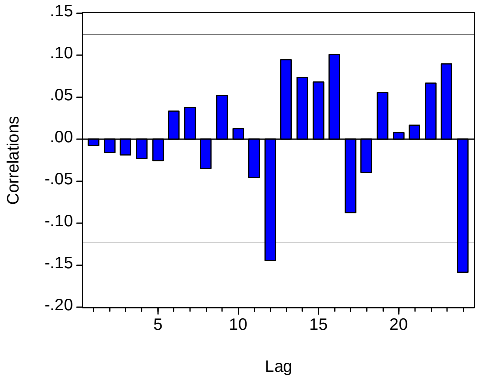


Figure xr9.13a Residual correlogram for Exercise 9.13 part (a)

(iv) The forecast of y_t for January is

$$\hat{y}_{T+1} = \hat{\delta} + \hat{\delta}_3 x_{T-2} + \hat{\theta}_1 y_T + \hat{\theta}_2 y_{T-1} + \hat{\theta}_3 y_{T-2}$$
$$= 0.4856 - 58.4292 \times 0.3 - 0.5227 \times (-129) - 0.2903 \times 85 - 0.1641 \times (-112)$$
$$= 44.08$$

Similarly the forecast of y_t for February is

$$\hat{y}_{T+2} = \hat{\delta} + \hat{\delta}_3 x_{T-1} + \hat{\theta}_1 \hat{y}_{T+1} + \hat{\theta}_2 y_T + \hat{\theta}_3 y_{T-1}$$
$$= 0.4856 - 58.4292 \times 0.26 - 0.5227 \times 44.08 - 0.2903 \times (-129) - 0.1641 \times 85$$
$$= -14.24$$

The forecast of y_t for March is

$$\hat{y}_{T+3} = \hat{\delta} + \hat{\delta}_3 x_T + \hat{\theta}_1 \hat{y}_{T+2} + \hat{\theta}_2 \hat{y}_{T+1} + \hat{\theta}_3 y_T$$
$$= 0.4856 - 58.4292 \times (-0.06) - 0.5227 \times (-14.24) - 0.2903 \times 44.08$$
$$- 0.1641 \times (-129)$$
$$= 19.80$$

Exercise 9.13(b) (continued)

(b) (v) The forecast of housing starts, in thousands of houses, for January 2006 is

$$\widehat{HOUSE}_{T+1} = \hat{y}_{T+1} + HOUSE_T = 44.08 + 2002 = 2046$$

Similarly, for February 2006,

$$\widehat{HOUSE}_{T+2} = \hat{y}_{T+2} + \widehat{HOUSE}_{T+1} = -14.24 + 2046.08 = 2032$$

For March 2006 it is

$$\widehat{HOUSE}_{T+3} = \hat{y}_{T+3} + \widehat{HOUSE}_{T+2} = 19.80 + 2031.84 = 2052$$

EXERCISE 9.14

(a) Testing the null hypothesis $H_0: \rho = 0$ against the alternative $H_1: \rho \neq 0$ we obtain the test statistic value *LM* = 4.383 with a corresponding *p* value of 0.0363. Since the *p* value is less than a significance level of 0.05, we reject the null hypothesis and conclude that the errors in this model are correlated.

(b) There are a number of possible ARDL models that could be chosen here. Given the relatively small number of observations, we have opted for the simplest one that eliminates first-order autocorrelation, namely an ARDL(1,0) model. Also, the coefficients of the extra terms included in ARDL(2,0) and ARDL(1,1) models were found to be insignificant. If you experiment with more lags you will find that an ARDL(4,1) model has a large number of significant coefficients. However, estimating such a large model with such a small sample is too ambitious. The estimated ARDL(1,0) model is

$$\widehat{\ln(UNITCOST_t)} = 2.5088 - 0.1862\ln(CUMPROD_t) + 0.6417\ln(UNITCOST_{t-1})$$
$$\text{(se)} \quad (1.2062) \quad (0.0787) \quad\quad\quad (0.2005)$$

The *LM* test for AR(1) errors yields a test value of $LM = 0.756$ with corresponding *p*-value of 0.3845, indicating that the correlation found in part (a) has been eliminated by the inclusion of $\ln(UNITCOST_{t-1})$.

(c) Using the notation

$$\ln(UNITCOST_t) = \delta + \delta_0\ln(CUMPROD_t) + \theta_1\ln(UNITCOST_{t-1}) + v_t$$

the lag weights are given by $\beta_s = \delta_0\theta_1^s$, from which we get the following estimates.

Lag weight	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$
estimate	– 0.1862	– 0.1195	– 0.0767	– 0.0492	– 0.0316	– 0.0203

These estimates suggest that, as cumulative production increases, most of the learning effect that reduces unit cost occurs immediately, but there is also a gradually declining learning effect that continues to reduce costs beyond the immediate period.

(d) The prediction of the natural logarithm of *UNITCOST* for 1971 is given by

$$\widehat{\ln(UNITCOST_{T+1})} = 2.5088 - 0.1862\ln(3800) + 0.6417\ln(16.41631)$$
$$= 2.7697$$

In 1970, $\ln(UNITCOST_T) = 2.7983$. Thus, in line with the learning model, we expect unit cost to be less in 1971 than in it was 1970.

EXERCISE 9.15

(a) With the trend specified as $t = 1, 2, \ldots, 111$, The least squares estimated equation is

$$\widehat{\ln(POW_t)} = -0.1708 + 0.0082t - 0.000037t^2 + 0.9365\ln(PRO_t)$$
$$\text{(se)} \quad (0.4147)\ (0.0005)\quad (0.000004)\quad (0.0899)$$

The positive sign for b_2 and the negative sign for b_3, and their relative magnitudes, suggest that the trend for ln(*POW*) is increasing at a decreasing rate. A positive b_4 implies the elasticity of power use with respect to productivity is positive. The residual correlogram is depicted in the figure below. There is strong evidence of autocorrelation with significant positive correlations exceeding the significance bound $1.96/\sqrt{111} = 0.186$ up to lag 7, and some significant negative correlations less than the negative bound $-1.96/\sqrt{111} = -0.186$ beyond lag 19.

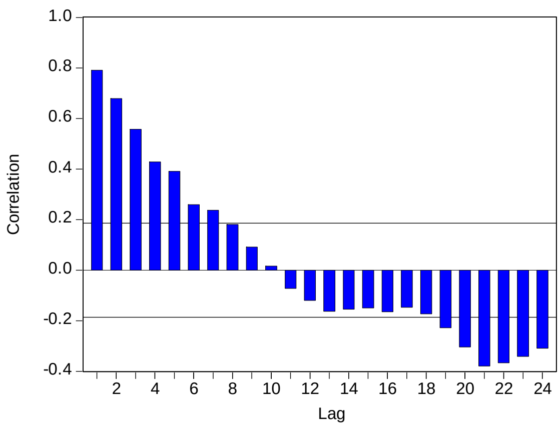


Figure xr9.15a Residual correlogram for Exercise 9.15, part(a)

(b) The estimated ARDL(1,1) model is

$$\widehat{\ln(POW_t)} = 0.3662 + 0.0016t - 0.000008t^2 + 0.9828\ln(PRO_t)$$
$$\text{(se)} \quad (0.2555)\ (0.0006)\quad (0.000003)\quad (0.1052)$$
$$-\,0.8768\ln(PRO_{t-1}) + 0.7966\ln(POW_{t-1})$$
$$(0.1110) \qquad\qquad (0.0584)$$

One lag of *POW* and one lag of *PRO* were used because the coefficients of the longer lags were not significantly different from zero. In addition, the residual correlogram in Figure xr9.15b suggests that autocorrelation has been largely eliminated with only r_5, r_6 and r_{21} statistically significant and even these values are relatively small.

Exercise 9.15(b) (continued)

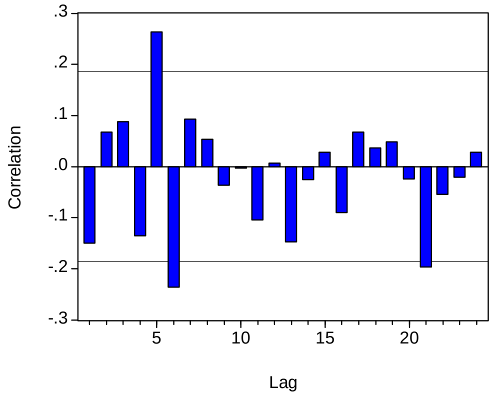


Figure xr9.15b Residual correlogram for Exercise 9.15, part(b)

(c) The *p*-values for testing the hypothesis $H_0: \beta_4 = 1$ are 0.4817 and 0.8704 for parts (a) and (b), respectively. We do not reject H_0 in both cases. Including more lags to correct for autocorrelation has led to a large change in the *p*-value, but the test decision is still the same.

(d) Denoting the coefficients of $\ln(PRO_t)$ and $\ln(PRO_{t-1})$ as δ_0 and δ_1, respectively, and that of $\ln(POW_{t-1})$ as θ_1, the lag weights can be shown to be

$$\delta_0,\ \theta_1\delta_0+\delta_1,\ \theta_1(\theta_1\delta_0+\delta_1),\ \theta_1^2(\theta_1\delta_0+\delta_1),\ \theta_1^3(\theta_1\delta_0+\delta_1),\ \ldots$$

The total multiplier is given by their sum

$$\begin{aligned}&\delta_0+\theta_1\delta_0+\delta_1+\theta_1(\theta_1\delta_0+\delta_1)+\theta_1^2(\theta_1\delta_0+\delta_1)+\theta_1^3(\theta_1\delta_0+\delta_1)+\cdots\\ &=\delta_0(1+\theta_1+\theta_1^2+\theta_1^3+\cdots)+\delta_1(1+\theta_1+\theta_1^2+\theta_1^3+\cdots)\\ &=\frac{\delta_0+\delta_1}{1-\theta_1}\end{aligned}$$

Using the estimates corresponding to these parameters the total multiplier is

$$\frac{\hat{\delta}_0+\hat{\delta}_1}{1-\hat{\theta}_1}=\frac{0.98278-0.87683}{1-0.79663}=0.5210$$

EXERCISE 9.16

(a) The least-squares estimated model without lags is

$$\widehat{\ln(POW_t)} = 0.4306 - 0.0399 D_t + 0.0092 t - 0.000041 t^2 + 0.8035 \ln(PRO_t)$$
$$\text{(se)} \quad (0.4728) \; (0.0162) \quad (0.0006) \; (0.000005) \quad (0.1030)$$

The estimated model including lags is

$$\widehat{\ln(POW_t)} = 0.5620 - 0.0130 D_t + 0.0021 t - 0.0000099 t^2 + 0.9543 \ln(PRO_t)$$
$$\text{(se)} \quad (0.2968) \; (0.0101) \quad (0.0007) \quad (0.0000038) \quad (0.1072)$$
$$- 0.8773 \ln(PRO_{t-1}) + 0.7806 \ln(POW_{t-1})$$
$$(0.1107) \qquad (0.0595)$$

(b) The *p*-value for the hypothesis test $H_0 : \delta_1 = 0$ for the model is 0.0152. Since it is less than the level of significance 0.05, we reject the null hypothesis and conclude that the dummy variable is statistically significant.

The *p*-value for the hypothesis test $H_0 : \delta_1 = 0$ for the model with lags is 0.2019. Since it is greater than the level of significance 0.05, we do reject the null and therefore cannot conclude that the dummy variable is significant.

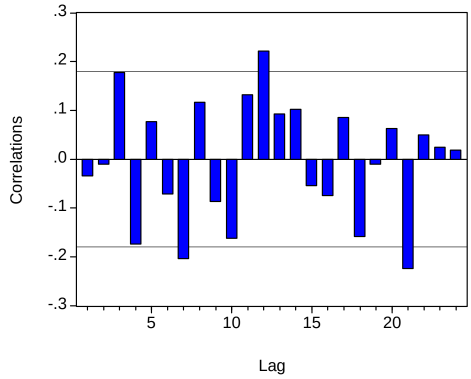
Thus, when we do not account for autocorrelation the structural change is statistically significant, but when we do correct for autocorrelation, the structural change is no longer significant. In general, these results suggest that, if we do not specify the correct lag structure, we can make misleading conclusions about the existence of structural change.

EXERCISE 9.17

(a) The estimated model is:

$$ROB_t = -5.6204 + 1.5614t + 0.5639ROB_{t-1}$$
$$\text{(se)} \qquad (0.2873) \quad (0.0769)$$

The correlogram of the residuals is shown below. The significance bounds are drawn at $\pm 1.96/\sqrt{117} = 0.181$. There are a few significant correlations at long lags (specifically at lag orders 7, 12 and 22), but they are relatively small. The spike at lag 12 could indicate a monthly seasonal effect.



(b) The predicted values in November and December are respectively:

$$\widehat{ROB}_{T+1} = -5.6204 + 1.5614 \times 119 + 0.5639 \times 431 = 423.2$$

$$\widehat{ROB}_{T+2} = -5.6204 + 1.5614 \times 120 + 0.5639 \times 423.2 = 420.4$$

The standard errors of prediction are:

$$\hat{\sigma}_1 = \hat{\sigma}_v = 36.713$$

$$\hat{\sigma}_2 = \sqrt{\hat{\sigma}_v^2\left(1+\theta_1^2\right)} = \sqrt{36.713^2\left(1+0.5639^2\right)} = 42.148$$

The confidence intervals are:

$$\widehat{ROB}_{T+1} \pm t_{(0.975,114)} \times \hat{\sigma}_1 = 423.2 \pm 1.981 \times 36.713 = (350,\ 496)$$

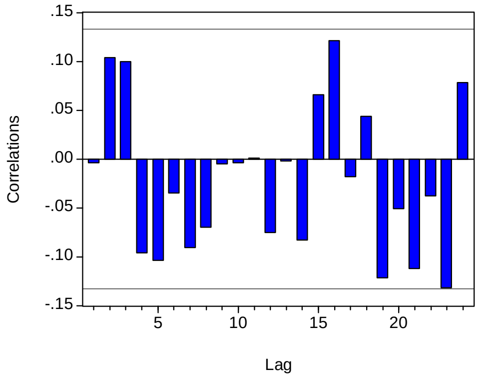
$$\widehat{ROB}_{T+2} \pm t_{(0.975,114)} \times \hat{\sigma}_2 = 420.4 \pm 1.981 \times 42.148 = (337,\ 504)$$

EXERCISE 9.18

(b) (i) The estimated ARDL model is

$$y_t = 0.2552 + 0.3685 x_t + 0.2299 x_{t-1}$$
$$\text{(se)} \quad (0.0543)(0.0455) \quad (0.0456)$$

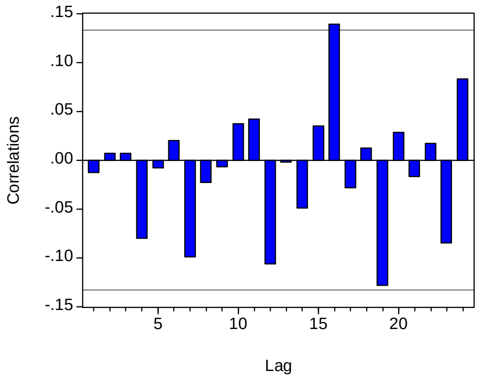
Only one lag of x_t was included because neither additional lags of x_t nor lags of y_t were statistically significant, with the exception of y_{t-4}. Including y_{t-4} would have unnecessarily complicated the model, particularly in light of the residual correlogram given below, which shows no significant autocorrelations. The significance bounds in this correlogram are at $\pm 1.96/\sqrt{215} = \pm 0.134$.



(ii) The estimated ARDL model is

$$z_t = -0.0493 + 1.2379 x_t + 1.0709 x_{t-1} - 0.1975 z_{t-1}$$
$$\text{(se)} \quad (0.2908)(0.2488) \quad (0.2549) \quad (0.0672)$$

An ARDL(1,1) model was specified because additional lags of both variables were not statistically significant, and they were also unnecessary to eliminate autocorrelation. All correlations in the below correlogram are small, with only that at lag 16 marginally significant.



Exercise 9.18 (continued)

(c) (i) For (b) (i) the lag weights for 8 quarters are:

$$\hat{\beta}_0 = 0.3685,\ \ \hat{\beta}_1 = 0.2299,\ \ \hat{\beta}_2 = \hat{\beta}_3 = \hat{\beta}_4 = \hat{\beta}_5 = \hat{\beta}_6 = \hat{\beta}_7 = \hat{\beta}_8 = 0$$

The lag weights beyond lag 1 are zero because there is no lagged dependent variable in the model. The estimated lag weights suggest that a temporary 1 unit increase in x_t will cause y_t to increase by 0.3685 at time *t*, and y_{t+1} to increase by 0.2299 at time $t+1$, with no changes in y from time $t+2$ onwards. In terms of the original variables, a 1% temporary increase in the growth of disposal personal income, will lead to an immediate increase in the growth of personal consumption of 0.3685%, and an increase of in the following quarter of 0.2299%, with no further changes in subsequent periods.

Using the notation

$$z_t = \delta + \delta_0 x_t + \delta_1 x_{t-1} + \theta_1 z_{t-1} + v_t$$

the lag weights for (b) (ii) for the first 8 quarters are given by the expressions

$$\hat{\beta}_0 = \hat{\delta}_0$$
$$\hat{\beta}_1 = \hat{\delta}_1 + \hat{\theta}_1\hat{\beta}_0$$
$$\hat{\beta}_s = \hat{\theta}_1\hat{\beta}_{s-1}, \qquad s > 1$$

Making these calculations yields

$$\hat{\beta}_0 = 1.2379, \quad \hat{\beta}_1 = 0.8264, \quad \hat{\beta}_2 = -0.1632, \quad \hat{\beta}_3 = 0.0322$$
$$\hat{\beta}_4 = -0.0064, \quad \hat{\beta}_5 = 0.0013, \quad \hat{\beta}_6 = -0.0002, \quad \hat{\beta}_7 = 0.0000, \quad \hat{\beta}_8 = -0.0000$$

These lag weights show the changes in growth of consumption of durable goods due to a temporary increase in income growth. They suggest that the current and one-quarter lagged effects are relatively large and positive, but after that the effects are relatively small and oscillate in sign, the oscillation being a consequence of the negative estimate for θ_1.

Exercise 9.18(c) (continued)

(ii) The total multiplier for part (b) (i) is

$$\sum_{s=0}^{1}\hat{\beta}_s = \hat{\beta}_0 + \hat{\beta}_1 = 0.3685 + 0.2299 = 0.5984$$

This result shows that the long run effect on *y* of a sustained 1-unit increase in *x* is 0.5984.

The total multiplier for part (b) (ii) is

$$\begin{aligned}\sum_{s=0}^{\infty}\hat{\beta}_s &= \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \cdots \\ &= 1.2379 + 0.8264 - 0.1632 + 0.0322 - 0.0064 + 0.0013 - 0.0002 + 0.0000 + \cdots \\ &= 1.9280\end{aligned}$$

This result can also be obtained from

$$\begin{aligned}\sum_{s=0}^{\infty}\hat{\beta}_s &= \hat{\beta}_0 + \hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \cdots = \frac{\hat{\delta}_0 + \hat{\delta}_1}{1 - \hat{\theta}_1} \\ &= \frac{1.2379 + 1.0709}{1 - (-0.1975)} \\ &= 1.9280\end{aligned}$$

It shows that the long run effect on *z* of a sustained 1-unit increase in *x* is 1.928.